3. Algebraic Structures / Expressions

MATERIALS NEEDED: 11 or more of each shape per group

The aim of this activity is to develop the ability to pick relevant elements and to recognise recurring patterns in a group of objects. We are going to define 'operations' on our group of five kinds of shapes, and will build a variety of algebraic structures. Based on the characteristics of said structures, we will ask pupils to complete a given sequence of objects or to choose a structure, which will enable us to solve a certain problem. The advantage of employing our shapes, hence of working with concrete finite groups of objects, is that we can define operations in such a manner that makes certain features 'visible', and highlight s the difference amongst the various algebraic structures.

Let us define following operations:

O.1
With this operation, our set of 5 shapes constitutes an abelian group* (also called a commutative group*), with the rhombus as the neutral element.

*A group in which the result of applying the group operation to two group elements does not depend on their order (the axiom of commutativity).

O.2
With this operation, our set of 5 shapes constitutes a commutative monoid* , with the rhombus as the neutral element.

*A monoid is an algebraic structure with a single associative binary operation and an identity element. A monoid whose operation is commutative is called a commutative monoid (or, less commonly, an abelian monoid). Commutative monoids are often written additively.