Pointy Fences

Task 181 ... Years 4 - 10

Content

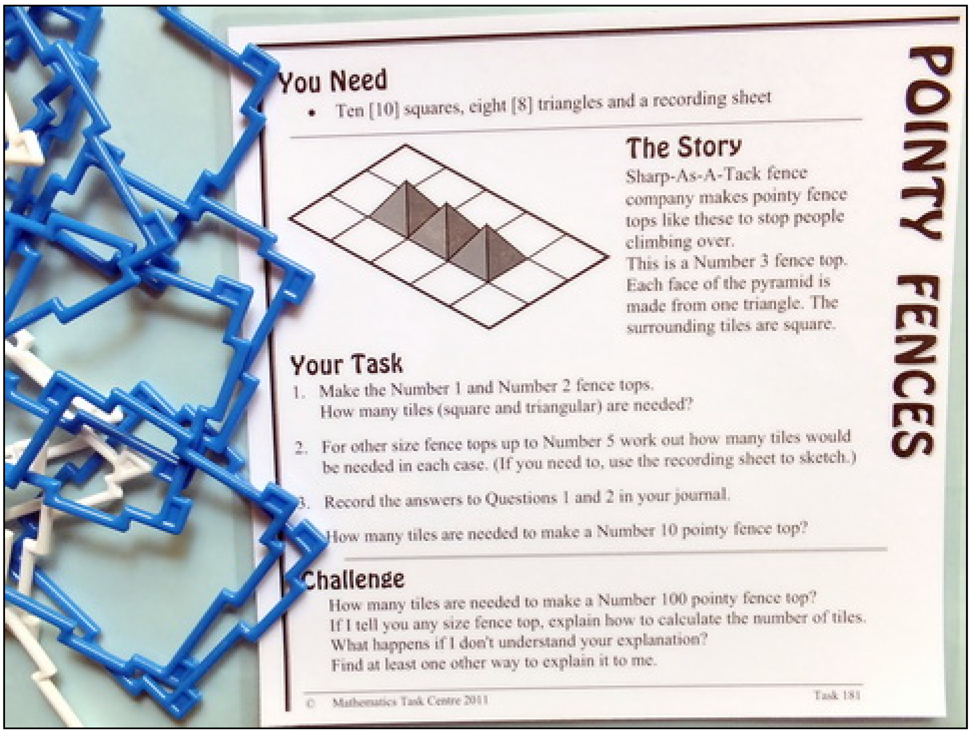
- algebra including:
  - concept of a variable
  - generalisation
  - substitution
  - solving linear functions
  - domain and range
  - equivalent algebraic expressions
- arithmetic, addition & subtraction
- arithmetic, multiplication & division
- equations, substitution & solution
- graphical representation
- mental arithmetic
- multiplication, array concept
- multiplication, calculations / times tables
- patterns, number
- patterns, visual

Summary

One hopes students haven't had too much experience with climbing over walls protected by spikes, but none-the-less the story shell gives the task a context that helps them imagine how the mathematics works. So, this visual and kinaesthetic challenge leads into a number pattern and the pattern encourages generalisation. Therefore this is another in the visual algebra series which includes 4 Arm Shapes, Garden Beds, Staircase, Double Staircase, Fold Up Houses and others. As with all these tasks, the way students 'see' the construction can lead to different ways of describing it, which in turn gives rise to equivalent algebraic expressions.

Materials

- 10 click-together squares and 8 click-together triangles
- Recording Sheet



Iceberg

A task is the tip of a learning iceberg. There is always more to a task than is

It is important to realise that the task is asking for the total number of shapes (N) in each case, but to calculate *recorded on the card.*

that students need to consider the two parts of the fence top, that is, the pyramids made from triangles and the square 'pathway' around them.

Fence tops 1 and 2 can be made with the equipment and students quickly succeed in discovering that if the number of points (P) is one, N = 12 and if P = 2, N = 18.

The key to generalising the problem is visualising sizes of fence top when materials aren't provided to make them, so Question 2 encourages that visualisation with the support of the drawing on the card and the Recording Sheet for sketching. Then students will discover that if:

- P = 3, N = 24
- P = 4, N = 30
- P = 5, N = 36

Question 3 encourages recording so that students have to reflect on their calculations and how to present them so they are understood. Perhaps they will sketch each size and add arrows and comments to explain, or perhaps they will use a table such as:

No of Points (P)	1	2	3	4	5
Number of Shapes (N)	12	18	24	30	36

Question 4, which asks for N given P = 10, might now be worked out in a number of ways, such as sketching or seeing the plus six pattern along the bottom row. The answer is that N = 66. However neither of these approaches is very efficient for P = 100, so if they haven't begun to do so already, students will need to be verbalising how they see the construction in order to efficiently calculate that N = 606 in this case.

So far, so good, but now comes the real challenge - explaining how to calculate N for any P and being able to do so in two different ways (at least) because this is what a mathematician has to do to check their work (see the Working Mathematically process). Possible generalisations are:

- Generalisation A
  There's two end bits and the middle. The middle is 2 squares for each pyramid and 4 triangles to make a pyramid. The end bits are 3 squares each. So to

find the total of shapes you multiply the number of points by 2 to get the squares and also multiply it by 4 to get the triangles. Then you add on the 6 for the ends.

- Generalisation B
  The strips with the points are always made from 6 shapes - 2 squares and 4 triangles. The end bits are always 6 squares. So to find the total of shapes you multiply the number of points by 6 and add 6 for the ends.
- Generalisation C
  You are only counting shapes. It doesn't matter which sort they are. The 6 end bits use up the same number of shapes as one of the pointy strips. There's 6 shapes in a pointy strip and 6 shapes in the end bit. So to find the total of shapes add 1 to the number of points and multiply all that by 6.
- Generalisation D
  Suppose the Sharp-As-A-Tack fence company starts with a grid of squares in three rows in one part of the factory, then they cut out the squares where the pyramids are going to go. In another part of the factory they make lines of pyramids the right length. Then they bring the pyramids to the grid with the holes and weld the pyramids in. Then, to find the total number of shapes, add 2 to the number of points and multiply by 3. Then subtract the number of points once for the squares that are cut out. Then add the fence top number multiplied by 4 because the pyramids go in.
- Generalisation E
  The simplest fence top needs 12 shapes - one strip of 6 made with 2 squares and a pyramid of 4 triangles, plus 3 on each end. After that you are really only adding a strip each time. So to find the total number of shapes, start with 12 then add 6 times one less than the number of points.

All these explanations (and no doubt others) are equally valid and you can almost picture the student in your class who presented each one. This process of oral explanation, followed by writing the words already spoken, is what helps to build a bridge to symbolic representation of the visualisation. In context, these generalisation lead to the following symbolic representation:

- Generalisation A
  N = 2 x P + 4 X P + 6

$= 2P + 4P + 6$

- <u>Generalisation B</u>
  N = 6 x P + 6
  $= 6P + 6$
- <u>Generalisation C</u>
  N = 6 x (P + 1)
  $= 6(P + 1)$
- <u>Generalisation D</u>
  N = 3 x (P + 2) - P + 4 X P
  $= 3(P + 2) - P + 4P$
- <u>Generalisation E</u>
  N = 12 + 6 X (P - 1)
  $= 12 + 6(P - 1)$

Students know exactly what the symbols mean and 'algebra' or rather algebraic symbolism now makes sense.

For teachers there are now several ways to extend:

- *Are you kids trying to tell me that each of these equations gives the same answer no matter what size pointy fence? Convince me!*
  (Substitution in equations)
- *Okay if they all give the same answer, can you show me how to change one expression into another. For example, I can see that A can easily become B. Can you show me how to turn the others into the same as B?*
  (Equivalent Algebraic Expressions)
- *If I tell you any correct number of shapes, can you tell me the number of points in the pointy fence?*
  (Solution of equations).
- *Can you tell me how many of those will be squares and how many will be triangles?*
- *I can't just choose any number to give you for the number of shapes, so how do I know which are the correct ones?*
  (Domain & range)
- *Make pairs of numbers that link the number of points with the number of shapes (P, N). Graph the pairs and tell me what you discover.*
  (Linear algebra, gradient, intercepts)

If you have Maths With Attitude Pattern & Algebra Years 7 & 8, it includes an Investigation Guide that explores these extensions.

Whole Class Investigation

Tasks are an invitation for two students to work like a mathematician. Tasks can also be modified to become whole class investigations which model how a mathematician works.

It is important to have equipment for each pair to start this investigation, otherwise it becomes just another textbook-like exercise. Many schools have 3D Geoshapes or Mini-Geofix which are perfect for making the pointy fences. The materials list above shows how many squares and triangles are needed for each pair. It is useful to have enough prepared packs in sandwich bags for each pair.

Begin the lesson by gathering the students around a Size 3 pointy fence you have made in advance. Also in advance, sketch the Size 3 on recording paper as in the diagram on the card, and scan or photograph it for projection.

Introduce the Sharp-As-A-Tack company and explain that we have been hired to find a way to explain to their employees how to know the number of shapes to use to make any size pointy fence. Invite pairs to make Size 1 and Size 2. Display your Size 3 drawing and ask them to draw Sizes 1 and 2.

Develop the lesson using the notes above as a guide. Round off by asking each person to produce a poster to be displayed in the factory so that the workers will always know how to calculate the number of shapes and why the rule works. The poster should also include how to calculate not just the number of shapes, but how many will be squares and how many will be triangles. (We would be happy to include photos of your students' posters in this cameo.)

At this stage, *Pointy Fences* does not have a matching lesson on Maths300, but Lesson 16, *Garden Beds* is very similar.

Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The *Pointy Fences* task is an integral part of:

- MWA *Pattern & Algebra* Years 5 & 6
- MWA *Pattern & Algebra* Years 7 & 8

Follow this link to Task Centre Home page.