Tetrahedron Triangles

Task 186 ... Years 4 - 10

Summary

This task links working with nets and looking for number patterns and it leads on to algebraic representation. Students have enough material to make the Size 1 Tetrahedron, which gives 3D form to the first diagram on the card. Then they have to deconstruct it to have enough material to build the Size 2. The only clue given for this is a diagram of one face. From here on the 3D object has to be imagined based on making one face (up to Size 4) and/or sketches of one face. It soon becomes clear that the total of triangles is four times the number in one face. But how do we count the triangles in one face?

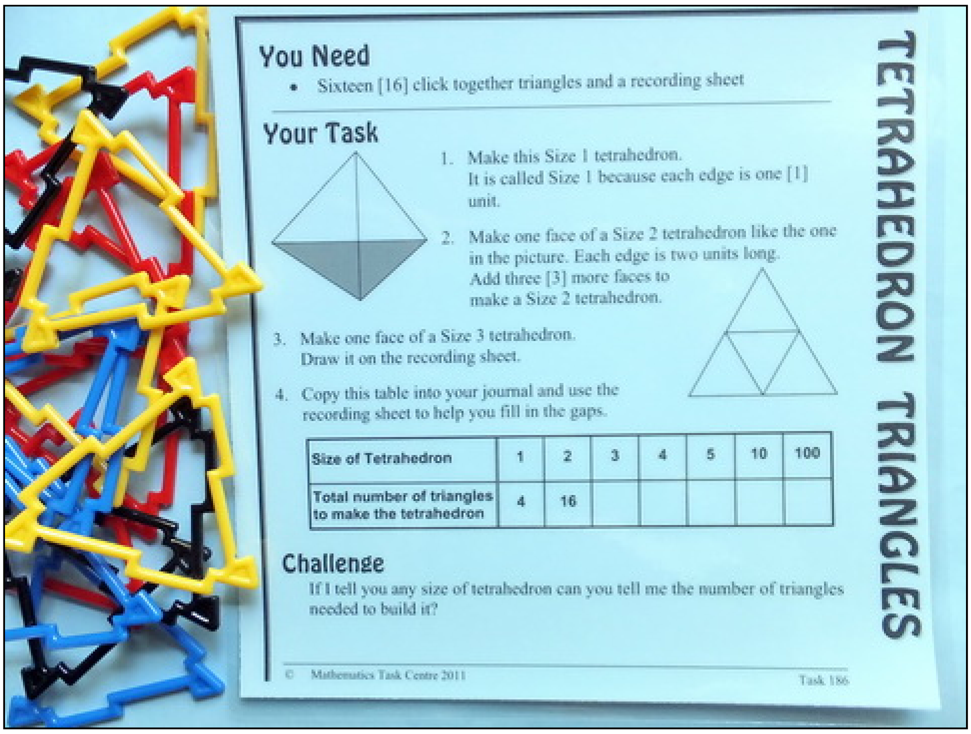
Tetrahedron Triangles also appears on the Picture Puzzles Pattern & Algebra A menu where the problem is presented using one screen, two learners, concrete materials and a challenge.

Materials

- 16 click together triangles
- Recording Sheet

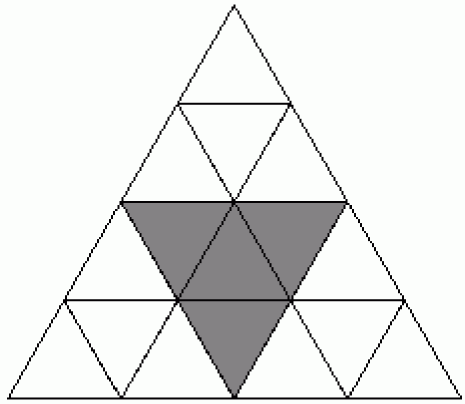
Content

- 2D representation of 3D objects
- algebra, concept of a variable / function
- algebra, generalisation in words & symbols
- algebra, quadratic
- equations, creating
- equations, substitution & solution
- graphical representation
- measurement, area
- mental arithmetic
- multiplication, calculations / times tables
- numbers, square
- numbers, triangle
- patterns, number
- patterns, visual
- sequences & series
- spatial perception, 2D or 3D



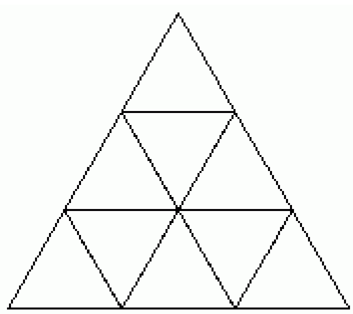
Iceberg

A task is the tip of a learning iceberg. There is always more to a task than is recorded on the card.



Question 1 is introducing the language of the card and making sure that 3D geometry involves a 3D model. Question 2 leads to the students building this partial net.
Folding up along the edges of the highlighted triangle creates the Size 2 Tetrahedron and its base is the highlighted triangle.

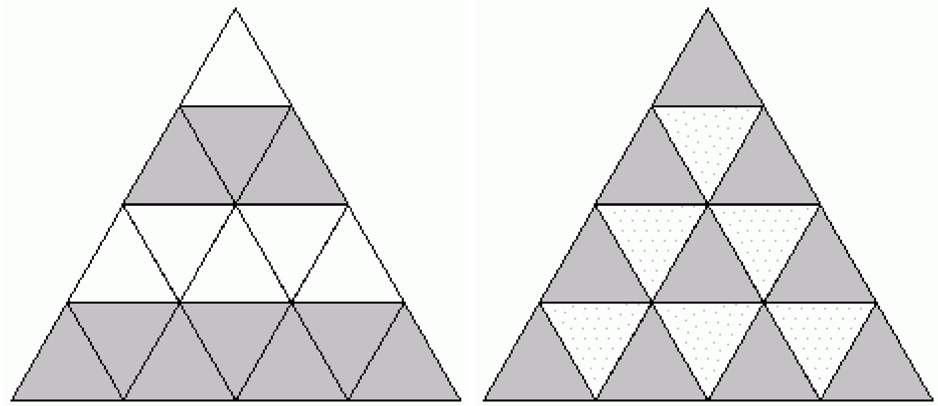
So far the students have discovered that to build the Size 1 Tetrahedron requires 4 unit triangles and the Size 2 requires 16. It may not be the unit usually chosen, but none-the-less the unit triangles provide a measure of the surface area of the tetrahedron. You might like to explore this concept with the students as they work with the task. A note in their journal would also be useful.



One face of Size 3 looks like this and counting the number of triangles is starting to get a little more complicated. There are nine, so the total needed is four times that, which is 36.

When it gets to counting Sizes 10 or 100, as in the table, drawing and counting one by one is not going to be efficient.

Students can make one face for tetrahedra up to Size 4 and some will notice the pattern 1, 4, 9, 16 and recognise it as the Side length squared, S^2. But students don't always see things the way we do (thank goodness) and might calculate the number of triangle in a face noticing one of these approaches.



The first of these suggests that the triangle is built by a sequence of odd numbers, so for one face of Size 10 we need, as Karen explained it:

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19
= (2 x 10) + 5 + (5 x 10) + (2 x 10) + 5
= (9 x 10) + 10
= 100

(Would you evaluate this the same way as Karen did? Would you have any discussion with Karen if she was in your class?)

The second visualisation sees that the point up triangles make a pattern of natural numbers and the point down triangles make the same pattern, but it stops one step before the point up one. In fact, each of these sets of triangles makes a triangle number pattern. So for one face of Size 10 we need, as Ray explained it:

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
= (5 x 11) + [(4 x 10) + 5]
= 55 + 45
= 100

(Would you evaluate this the same way as Ray did? Would you have any discussion with Ray if he was in your class?)

Whichever way they find their generalisation is valid. However, the simplest algebraic form is:

$$N = 4S^2$$

where N is the number of unit triangles needed.

Extensions

One challenge question has been asked on the card, but the existence of the generalisation provides opportunity to ask other questions such as:

- If I tell you the total number of unit triangles, can you tell me the size of the tetrahedron? Can I choose any number for this question to work? Explain.
- What happens if we make a graph of ordered pairs (S, N)?
- If we had to make a formula using Karen's way or Ray's way, what would it look like?
- Suppose someone wanted to build a sequence of Size 1 *and* Size 2 *and* Size 3 *and* Size 4 *and* Size 5 and ... so on. Can you work out the total number of triangles needed to build the sequence to Size N?
  (This just lifted the task to senior level, because the answer is going to involve summing square numbers.)

Whole Class Investigation

Tasks are an invitation for two students to work like a mathematician. Tasks can also be modified to become whole class investigations which model how a mathematician works.

For the fullest and most efficient experience as a whole class investigation you need click together triangles from either 3D Geoshapes, or MiniGeofix. Either allows easy building of the triangle nets and easy folding into three dimensions. MiniGeofix takes up less space on the table. The alternative is to use the Recording Sheet, scissors and sticky tape. You might use groups of 4 and start with something like:

> *Today we are going to connect different content areas of mathematics in the one investigation. We will be connecting geometry, number and algebra and even a little measurement. The first thing I want each person in the group to do is make (cut out) an equilateral triangle that has 3 triangle lengths along each side.*

Allow time for this then...

> *Now I want each group of 4 to join their triangles together to make an object called a tetrahedron.*

Now set the challenge and as the groups work, pause for discussion as appropriate and record key discoveries on the board. This will serve as a model for the students' journal work, which will come later.

If I tell you any size tetrahedron, I want you to be able to tell me the number of triangles to make it.

From here, taking into account student suggestions lead the mathematics through the work outlined above.

At this stage, *Tetrahedron Triangles* does not have a matching lesson on Maths300.

Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The *Tetrahedron Triangles* task is an integral part of:

- MWA *Pattern & Algebra* Years 9 & 10

Follow this link to Task Centre Home page.