How Many Triangles?

Task 48 ... Years 4 - 10

Summary

Build a new equilateral triangle from the Size 1 equilateral triangles supplied. Many patterns and generalisations then develop depending on what you count. The simplest is to just consider the relationship between the size of the new triangle and the number of Size 1 pieces which make it up. The problem becomes more complex if you look into the new size you create to look for triangles of any size within it. For example, a Size 3 has Size 2s within it as well as Size 1s. And then, what happens if we fold up these triangles to make tetrahedrons?
This task is similar to, but more complex than, Task 108, How Many Squares?.

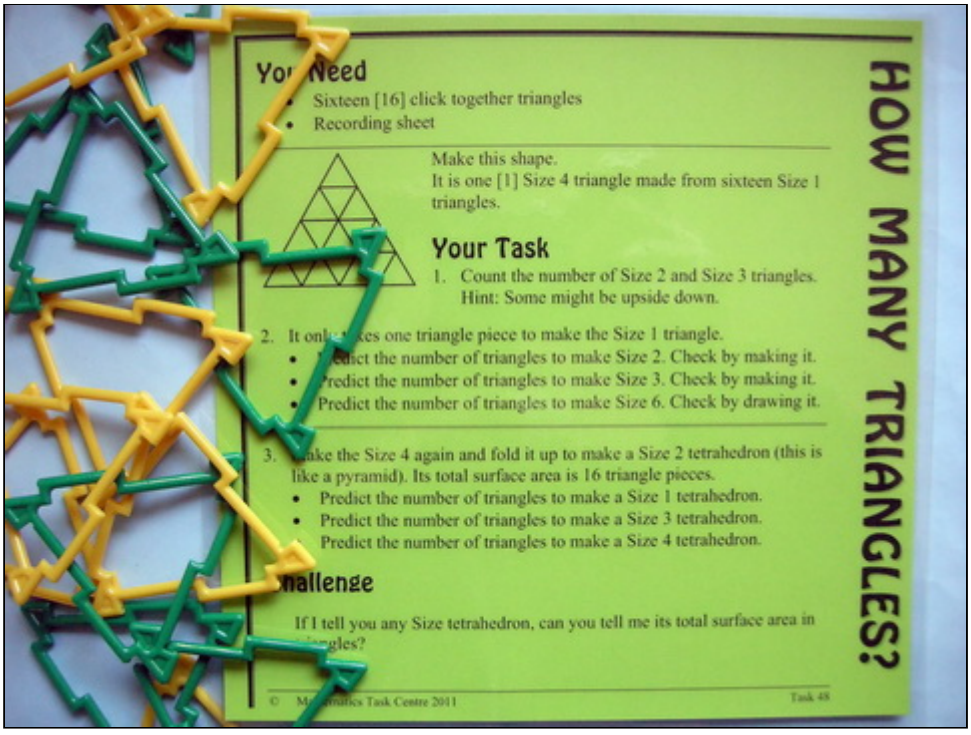
How Many Triangles also appears on the Picture Puzzles Pattern & Algebra A menu where the problem is presented using one screen, two learners, concrete materials and a challenge.

Materials

- At least 16 triangles - to explore the 3D extension it is best to have the click together ones shown (These are 3d Geoshape triangles - see 'Other Resources' in Resources & Ordering)
- Recording Sheet

Content

- number patterns
- square numbers
- triangle numbers
- algebraic generalisation in words and symbols
- graphing parabolic functions
- spreadsheet application



Iceberg

A task is the tip of a learning iceberg. There is always more to a task than is recorded on the card.

The first level of investigation - just counting unit triangles within the bigger one - shows that the number of unit triangles required to make Size N is N^2.

Therefore, if in the second part of the card the word triangle is interpreted as unit (Size 1) triangle, the formula required in the last part of the card is:

$$\text{Number of unit triangles} = 4 \times (\text{Size of tetrahedron})^2$$
$$T = 4S^2$$

Counting triangles at the more complex level of looking for all the size triangles (point up or point down) within a given size requires careful data collection to produce:

E_N = large equilateral triangle Size N
S_M = small equilateral triangle of Size M within it.

	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
S_1	1	4	9	16	25	36	49	64
S_2		1	3	7	13	21	31	43
S_3			1	3	6	11	18	27
S_4				1	3	6	10	16
S_5					1	3	6	10
S_6						1	3	6
S_7							1	3
S_8								1
Total	1	5	13	27	48	78	118	170

The link between E_N and the total of triangles is not straightforward. Even finding a link between E_N and triangles of particular size within it, say S_2, is not easy, although graphing the ordered pairs will suggest a quadratic relationship, which could be checked with a difference table. Further, students familiar with spreadsheets could use the Add Trendline tool to reveal the equation. Graphic calculators would be another approach.

However, these technologies don't explain why a rule exists. The answer to that begins to come clear as each line in the table above is split into Point Up and Point Down triangles. Then, the Triangle Numbers, which are tantalisingly hinted at in the last few rows of the table reveal themselves in two ways in each of the other rows.

Whole Class Investigation

Tasks are an invitation for two students to work like a mathematician. Tasks can also be modified to become whole class investigations which model how a mathematician works.

There is sufficient investigation in the first part of the card to make it only necessary to have 2D triangles, but if you have a set of the click together ones, so much the better. You can easily make sheets of unit triangles in software such as Word. The students begin the lesson by making their own materials. If you print on card and laminate before cutting you will have a set which will last several years. It could also be useful to make some large size unit triangles (say 10cm sides) to use as a community model on the floorboard to focus discussion.

This particular investigation would work well directed by an Investigation Guide. Prepare one based on the ideas in the iceberg above to lead your students in the appropriate direction for their age and experience.

At this stage, *How Many Triangles?* does not have a matching lesson on Maths300.

Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The *How Many Triangles?* task is an integral part of:

- MWA *Pattern & Algebra* Years 5 & 6
- MWA *Pattern & Algebra* Years 9 & 10

Follow this link to Task Centre Home page.