## Squares Around Squares

Task 24 ... Years 4-10

## Summary

This task embodies the mathematician's question, Can I check it another way?. Mini-squares are linked together to build larger squares in a pattern of alternating colours. The focus of the problem is on seeing and explaining the number of mini-squares used as the pattern grows. The overall problem is:

Given any size large square built in this way, predict, and justify in at least two ways:

## Materials

- 24 mini-squares of one colour and 12 of another
- Recording sheet


## Content

- number patterns
- generalisation of number patterns
- visual and symbolic representation of generalisations
- linear algebra
- quadratic algebra
- difference between two squares
- the total number of mini-squares used.
- the number of mini-squares of each colour.
- the number of mini-squares in the outside border.



## Iceberg

A task is the tip of a learning iceberg. There is always more to a task than is recorded on the card.

Students will have success with the closed questions above the double line on the card; they are really just being asked to make and count. Many find it unnecessary to actually link the squares - just laying them out in the pattern is enough.

Of course, a mathematician would record the counts, perhaps in a table:

## One Square Start

| SIZE | Total |  | Col. A |  | Col. B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 0 | Border |  |
| 3 | 9 | 1 | 8 | - |  |
| 5 | 25 | 17 | 8 | 16 |  |

## Two Square Start

| SIZE | Total |  | Col. A |  | Col. B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 4 | 0 | Border |  |
| 4 | 16 | 4 |  | 12 | - |
| 6 | 36 | 24 | 12 | 20 |  |
| - | - | - | - | - |  |

and this might encourage exploring the next size with a diagram, since there are not sufficient pieces to build it. Perhaps predict first and then check by drawing.


Whether or not the students take on The Challenge, the iceberg is in seeing and explaining how the actual construction of the squares around squares can be used to calculate the numbers in the tables. For example, consider only the question of the number of tiles in the outer border. Five ways students could look at this subproblem are:


But then again, the student might see the outer border as what remains when a 'doughnut' square is taken away from the outer square, ie: the difference between two squares. In that visualisation:
$B=S^{2}-(S-2)^{2}$
and given the standard textbook rule for finding the difference between two squares this becomes:
$B=[S-(S-2)][S+(S-2)]$
which of course evaluates to $B=4 S-4$ as do all the equations above, but how can we 'see' this classic expansion in a diagram?

## Whole Class Investigation

To convert this task to a whole class lesson you need lots of squares of some sort. If you don't have these MiniGeoshapes, square tiles, common in many schools, will do the job, as will wooden or plastic cubes. Geoboards
investigations which model how a mathematician works.
with different colour rubber bands can also be used. In addition square grid paper will be very useful for recording, but it is not a substitute for making some of the smaller size squares around squares.

At this stage Squares Around Squares does not have a matching Maths300 lesson.

## Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The Squares Around Squares task is an integral part of:

- MWA Pattern \& Algebra Years 9 \& 10

