Triangles Around Triangles

Task 42 ... Years 4 - 10

Summary

This task embodies the mathematician's question, *Can I check it another way?*. Mini-triangles are linked together to build larger triangles in a pattern of alternating colours. The focus of the problem is on seeing and explaining the number of mini-triangles used as the pattern grows. The overall problem is:

Given any size large triangle built in this way, predict, and justify in at least two ways:

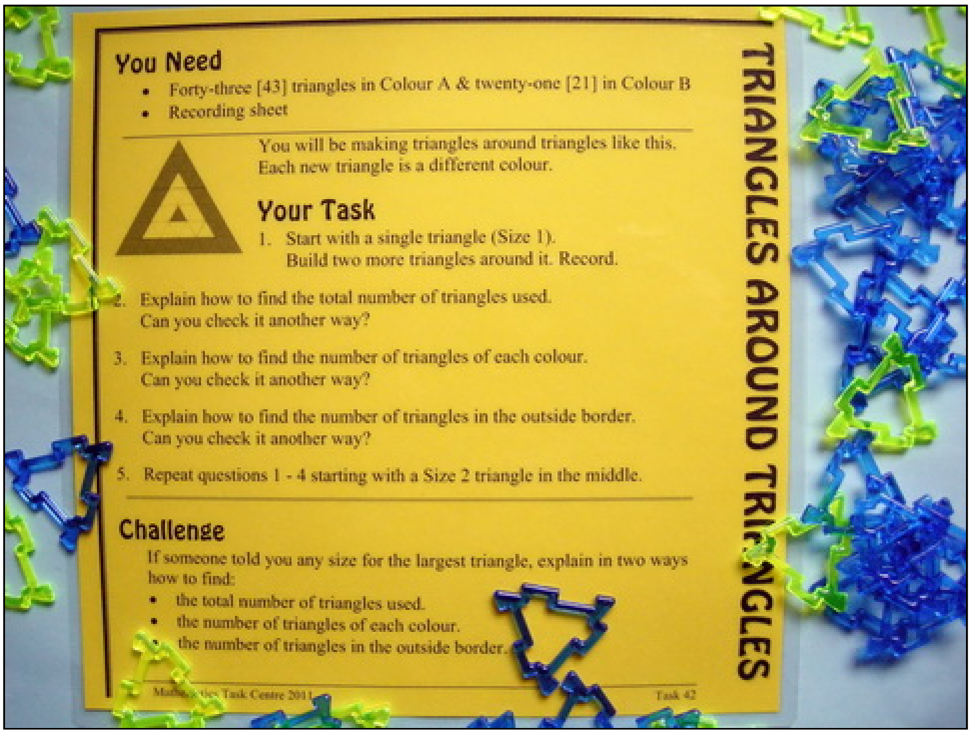
- the total number of mini-triangles used.
- the number of mini-triangles of each colour.
- the number of mini-triangles in the outside border.

Materials

- 43 mini-triangles of one colour and 21 of another
- Recording sheet

Content

- number patterns
- generalisation of number patterns
- visual and symbolic representation of generalisations
- linear algebra
- quadratic algebra
- difference between two squares

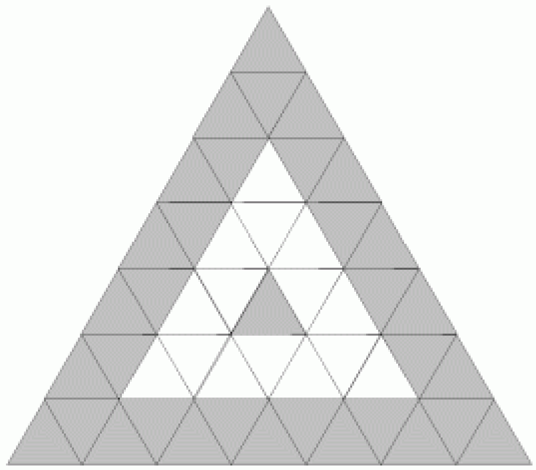


Iceberg

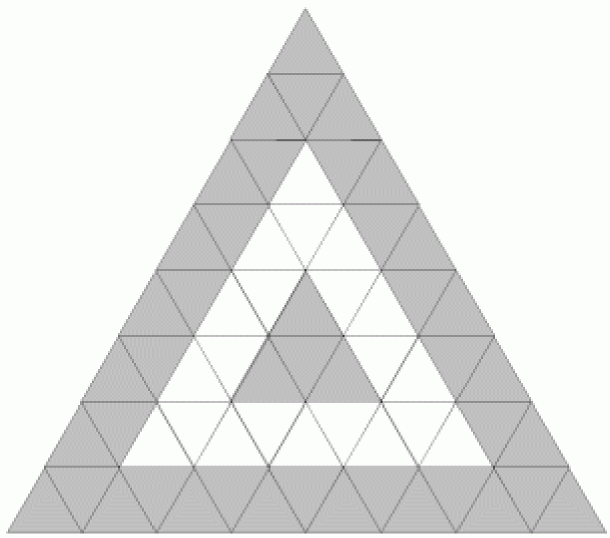
A task is the tip of a learning iceberg. There is always more to a task than is recorded on the card.

Students will have success with the closed questions above the double line on the card; they are really just being asked to make and count. Many find it unnecessary to actually link the triangles - just laying them out in the pattern is enough.

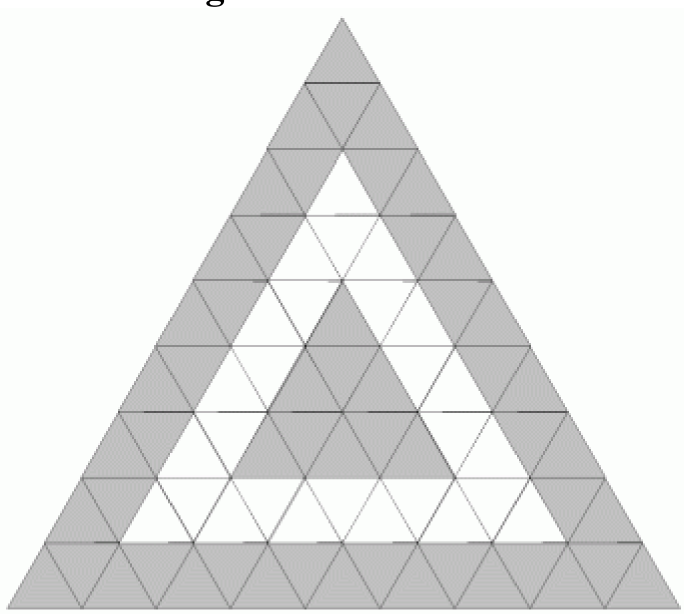
One Triangle Start



Two Triangle Start



Three Triangle Start



Of course, a mathematician would record the counts, perhaps in a table:

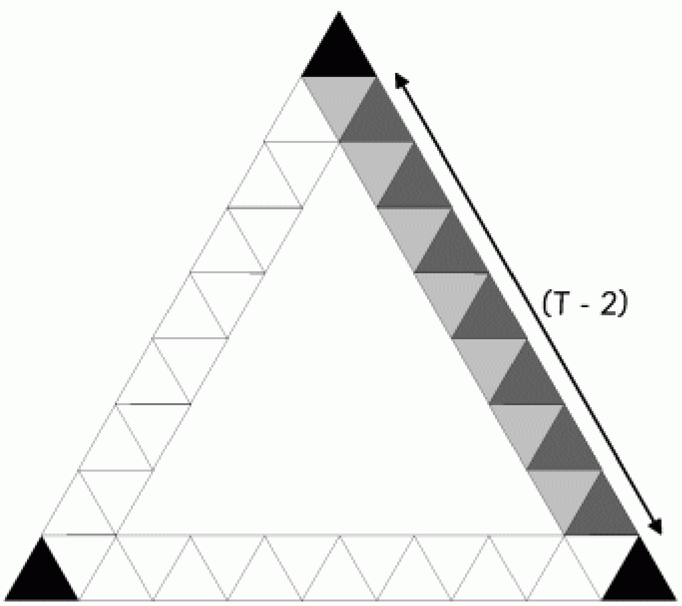
One Triangle Start

SIZE	Total	Col. A	Col. B	Border
1	1	1	0	__
4	16	1	15	15

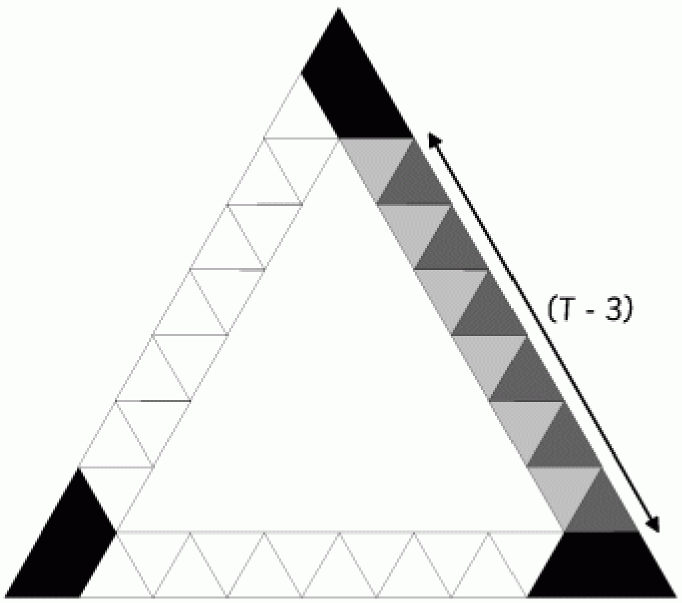
7	49	34	15	33
—	—	—	—	—

and similarly for the other starting sizes. Tabulating might encourage exploring the next size with a diagram, since there are not sufficient pieces to build it. Perhaps predict first and then check by drawing.

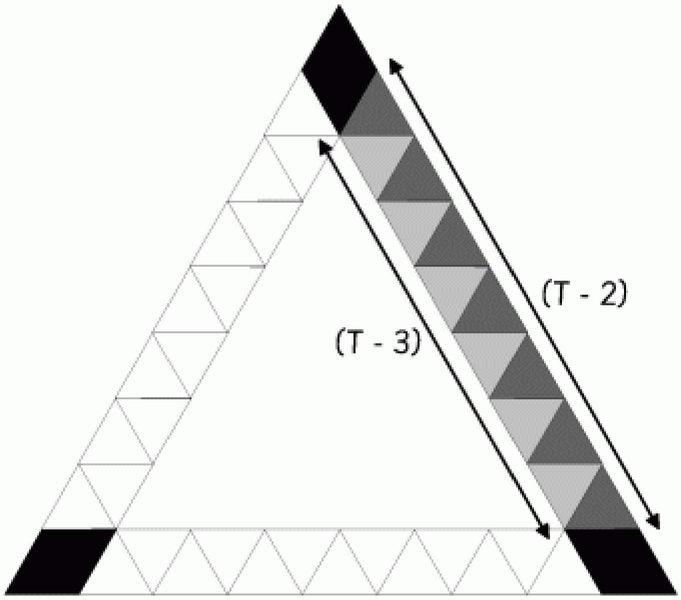
Whether or not the students take on The Challenge, the iceberg is in seeing and explaining how the actual construction of the triangles around triangles can be used to calculate the numbers in the tables. For example, consider only the question of the number of tiles in the outer border. Four ways students could look at this sub-problem are:



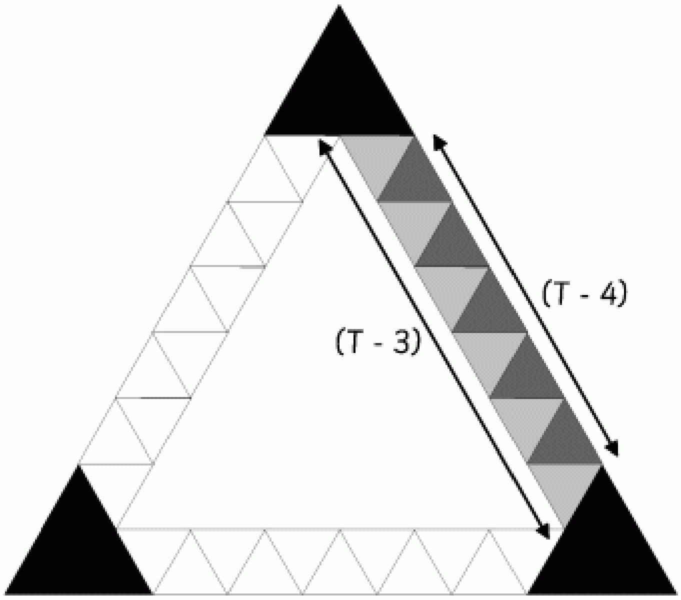
... B = 3[2(T - 2)] + 3)



... B = 3[2(T - 3)] + 3 x 3

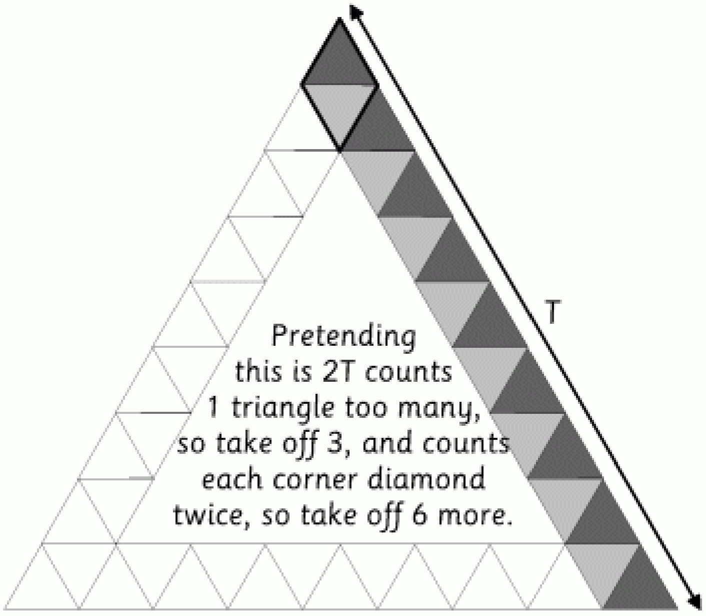


... $B = 3(T - 2) + 3(T - 3) + 3 \times 2$



... $B = 3(T - 4) + 3(T - 3) + 3 \times 4$

But then again, the student might see the outer border as what remains when a 'doughnut' triangle is taken away from the outer triangle, ie: the difference between two squares. In that visualisation:
$B = T^2 - (T - 3)^2$
and given the standard textbook rule for finding the difference between two squares this becomes:
$B = [T - (T - 3)]\ [T + (T - 3)]$
which of course evaluates to $3(2T - 3) = 6T - 9$, $T > 1$ as do all the equations above, but how can we 'see' this classic expansion in a diagram?



Whole Class Investigation

Tasks are an invitation for two students to work like a mathematician. Tasks can also be modified to become whole class investigations which model how a mathematician works.

To convert this task to a whole class lesson you need lots of triangles of some sort. If you don't have these Mini-Geoshapes, you may be able to have triangle tiles made by students as part of their craft curriculum. Alternatively you could prepare a sheet of triangle tiles and print for the students to cut up. A mark added to one side of the paper would represent the second colour. Triangle grid paper will be very useful for recording, but it is not a substitute for making some of the smaller size triangles around triangles.
Note: See Sphinx & Algebra 1 (Page 6, Sphinx Album) for an example of how cutting up paper tiles can lead to extensive student engagement. In this case the shape is much more complicated to cut than a tiling of equilateral triangles.

At this stage *Triangles Around Triangles* does not have a matching lesson on Maths300.

Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The *Triangles Around Triangles* task is an integral part of:

- MWA *Pattern & Algebra* Years 9 & 10

Follow this link to Task Centre Home page.