Change

Task 165 ... Years 2 - 10

Summary

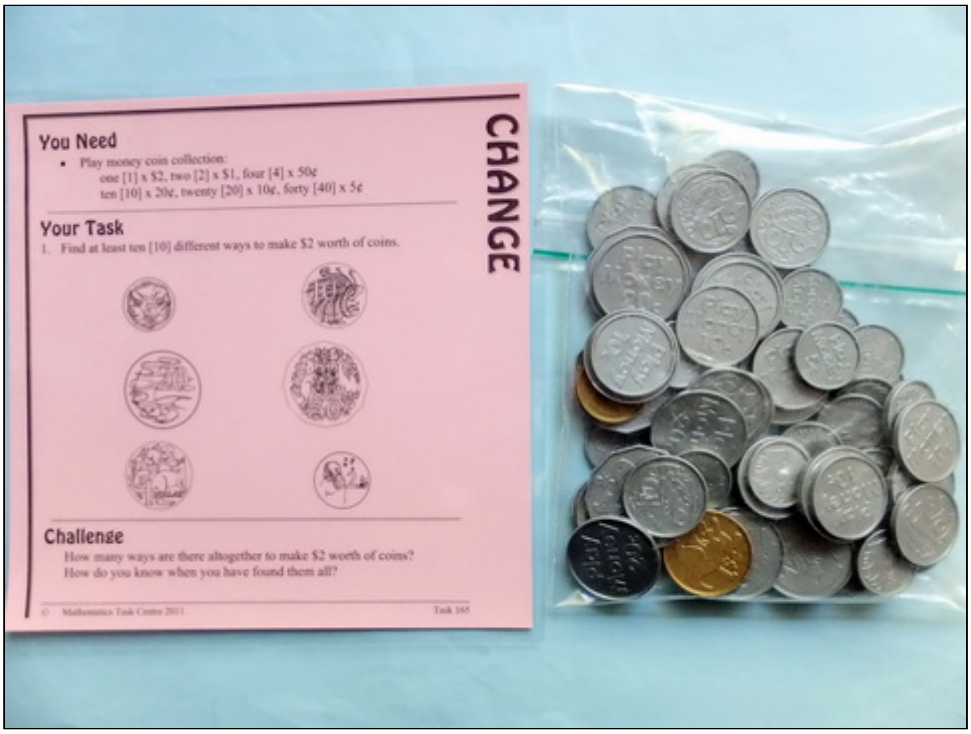
The tasks begins with a challenge any student can achieve, that of finding a few ways to make a total of $2 from coins. However, the challenge of finding *every* possible combination of coins that could make two dollars requires more effort. It encourages the strategies of breaking the problem into parts and trying every possible case. The challenge could be tackled with pencil and paper, and perhaps it will be by some students, but it is much more engaging, at least in the first instance, for many students to have the play money available to physically make substitutions between coins.

Materials

- Play money coins: 1 x $2, 2 x $1, 4 x 50¢,
  10 x 20¢, 20 x 10¢, 40 x 5¢

Content

- arithmetic, addition / subtraction
- arithmetic, multiplication / division
- counting
- money
- patterns, number
- reasoning



Iceberg

A task is the tip of a learning iceberg. There is always more to a task than is recorded on the card.

One way to record the investigation is as a table:

Solution No.	$2	$1	50¢	20¢	10¢	5¢	No. of Coins
1	1	-	-	-	-	-	1
2	-	2	-	-	-	-	2
3	-	1	2	-	-	-	3
4	-	1	1	2	1	-	5

5	-	1	1	2	-	2	6
6	-	1	1	1	3	-	6
7	-	1	1	1	2	2	7
8	-	1	1	1	1	4	8
9	-	1	1	-	5	-	7
10	-	1	1	-	4	2	8
11	-	1	1	-	3	4	9
12	-	1	1	-	2	6	10
13	-	1	1	-	1	8	11
14	-	1	1	-	-	10	12
15	-	1	-	5	-	-	6
16	-	1	-	4	2	-	7
17	-	1	-	4	1	2	8
18	-	1	-	4	-	4	9
19	-	1	-	3	4	-	8
20	-	1	-	3	3	2	9
21	-	1	-	3	2	4	10
22	-	1	-	3	1	6	11
23	-	1	-	3	-	8	12
24	-	1	-	2	6	-	9
25	-	1	-	2	5	2	10
26	-	1	-	2	4	4	11
27	-	1	-	2	3	6	12
28	-	1	-	2	2	8	13
29	-	1	-	2	1	10	14
30	-	1	-	2	-	12	15
31	-	1	-	1	8	-	10
32	-	1	-	1	7	2	11
33	-	1	-	1	6	4	12
34	-	1	-	1	5	6	13
35	-	1	-	1	4	8	14
36	-	1	-	1	3	10	15
37	-	1	-	1	2	12	16
38	-	1	-	1	1	14	17
39	-	1	-	1	-	16	18
40	-	1	-	-	10	-	11
41	-	1	-	-	9	2	12
42	-	1	-	-	8	4	13

43	-	1	-	-	7	6	14
44	-	1	-	-	6	8	15
45	-	1	-	-	5	10	16
46	-	1	-	-	4	12	17
47	-	1	-	-	3	14	18
48	-	1	-	-	2	16	19
49	-	1	-	-	1	18	20
50	-	1	-	-	-	20	21

Well that's the end of all the possibilities for using $2 coins and $1 coins. But it's not the end of all the possibilities. What happens if you don't use any $2 coins or $1 coins? Can you still make up $2 worth of change?

Clearly reasoning is the focus of this task. The value of the money drifted to the background long ago. Inexperienced students can't be expected to begin, let alone complete, this type of reasoning without support. The shading in the table suggests subsets of challenges that can be chosen to extend each pair appropriately. These sub-challenges can be offered verbally, or you can cut and paste the screen information into a word processor and turn it into a table, or tables, that become the basis of an Investigation Guide - a set of challenges that draw the students further into the iceberg.

There will be some students (and teachers) who will accept the challenge of completing the table from where it finishes above. Then you will know how many solutions there are - and you will know you have found them all because of the number patterns in the table, particularly in the **No. of Coins** column.

Whole Class Investigation

Tasks are an invitation for two students to work like a mathematician. Tasks can also be modified to become whole class investigations which model how a mathematician works.

Play money costs almost as much as the face value of the coins, so it is unlikely that you will have enough to make sets for each pair to use in a whole class investigation. It is possible to find print masters for the coins, but to produce sets this way will be costly on someone's time. However, with the set in the task, and relying on students' familiarity with money, it is still possible to build a whole class lesson. You will also need a spreadsheet version of the table above displayed on your screen. Use Window/Freeze Panes to keep the column headings in view. Students will need 1cm square graph paper as their recording tool.

Gather students at a central table in a fish bowl style and spread out the coins. Ask each of the students at the table to select coins to make a total of $2. Ask a scribe to record the various collections and the initials of the creator.

> *Let's put all the coins back in the centre. Could anyone make a total of $2 a different way?*

Encourage two or three more responses and ask the scribe to record these also.

> *In what way can we make $2 with the largest number of coins?*
> *In what way can we make $2 with the smallest number of coins?*

Ask students to return to their tables to find 3 ways of their own to make $2 and record them in their journal with appropriate heading and date.

> *We are learning to work like mathematicians, so what question might a mathematician ask as a result of playing around with this problem - the problem of ways of making $2 - like we have been?*

Encourage the question of how many ways altogether and discuss strategies that might be used. Lead the students towards making a table and breaking the problem into parts. Through discussion, begin to create the table above in the spreadsheet and ask students to copy it onto their graph paper. As the investigation continues all groups will be contributing to the various sections, but later each group might be asked to complete a particular part.

> *Group A you investigate all the possibilities for not using $2 or $1 coins but using 3 x 50¢ coins.*
> *Group B you investigate all the possibilities for not using $2 or $1 coins but using 2 x 50¢ coins.*
> *Group C you investigate all the possibilities for not using $2 or $1 coins but using 1 x 50¢ coins.*
> *Group D you investigate all the possibilities for not using $2 or $1 or 50¢ coins, but using...*

Continuing in this way the class will be able to find all the possible ways. Don't forget to congratulate them on working like mathematicians. This is definitely a non-trivial challenge.

At this stage, *Change* does not have a matching lesson on Maths300.

Is it in Maths With Attitude?

Maths With Attitude is a set of hands-on learning kits available from Years 3-10 which structure the use of tasks and whole class investigations into a week by week planner.

The *Change* task is an integral part of:

- MWA *Number & Computation* Years 5 & 6

Follow this link to Task Centre Home page.